Algebra 2, Quarter 3, Unit 3.1

Determining Inverse Functions

Overview

Number of instructional days: 5 (1 day = 45–60 minutes)

Content to be learned

- Solve equations of the form $f(x) = c$ for simple functions f that have an inverse.
- Write an expression for the inverse of equations.
- Realize that exponential and logarithm functions are inverses.

Mathematical practices to be integrated

Reason abstractly and quantitatively.

- Determine if two equations are inverses.
- Find the inverse of an exponential function.

Attend to precision.

- Explain the connection between exponential and logarithm functions.
- Determine the relationship between functions and inverses.

Look for and make use of structure.

- Use function and inverse function notation for equations.
- Perform a composition of inverse functions.

Essential questions

- How would you use inverse functions in a real-life situation?
- Why are exponential and logarithm functions inverses?

Written Curriculum

Common Core State Standards for Mathematical Content

Building Functions	F-BF

Build new functions from existing functions

F-BF.4 Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.

F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Common Core State Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students learned understanding of addition and subtraction. During the first grade, students represented and solved problems involving addition and subtraction. They applied the properties of operations and worked with addition and subtraction equations. Second-grade students continued to represent and solve problems involving addition and subtraction and began working with groups of objects to gain foundations for multiplication. In third grade, students represented and solved problems involving multiplication and division. They understood the properties of multiplication and the relationship between multiplication and division. They solved problems involving the four operations and identified and explained patterns in arithmetic. The students developed understanding of fractions as numbers.

Students continued to use the four operations with whole number to solve problems during fourth grade. They gained familiarity with factors and multiples and generated and analyzed patterns. Students extended their understanding of fraction equivalence and ordering. They built fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Students understood decimal notation for fractions and compared decimal fractions. During the fifth grade, students wrote and interpreted numerical expressions and analyzed patterns and relationships. They used equivalent fractions as a strategy to add and subtract fractions. To multiply and divide fractions, students applied and extended previous understandings of multiplication and division. Students applied and extended previous understandings of arithmetic to algebraic expressions during sixth grade. They reasoned about and solved one-variable equations and inequalities. Students represented and analyzed quantitative relationships between dependent and independent variables. Application and extension of previous understanding of multiplication and division was used to divide fractions.

During the seventh grade, students applied and extended previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers. They used properties of operations to generate equivalent expressions. They solved real-life and mathematical problems using numerical and algebraic expressions and equations. In the eighth grade, students knew that there are numbers that are not rational and approximated them by rational numbers. The students worked with radicals and integer exponents. They understood the connections between proportional relationships, lines, and linear equations. They defined, evaluated, and compared functions. Students used functions to model relationships between quantities. In Algebra 1, the students interpreted the structure of expression and wrote expressions in equivalent forms to solve problems. They created equations that derived number or relationships. They solved equations and inequalities.

Current Learning

Students solve equations and write the inverse of the equations. Algebra 2 pre-AP students learn that exponential functions and logarithmic functions are inverses.

Future Learning

Students will apply the writing of inverses in their fourth-year math course and future college courses. Students will use this knowledge and these skills in professions such as financial managers, budget analysts, ranchers, and logging workers.

Additional Findings

Building new functions from existing functions is considered an additional cluster. This cluster will not be tested on the end-of-course test, but it will be needed for future math courses.

Algebra 2, Quarter 3, Unit 3.2

Connecting Arithmetic and Geometric Sequences and Series with Linear and Exponent Functions

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Build algebraic expressions
- Use equivalent algebraic expressions to write equations.
- Write equations as functions using function notation.
- Construction functions to model lines, quadratic and exponential equation in a situations from table, graphs, word problems, and real-life situations.
- Write functions for arithmetic and geometric sequences.
- Write recursive and explicit functions.

Mathematical practices to be integrated

Model with mathematics.

- Apply knowledge of arithmetic sequences and functions to solve real-life problems.
- Model problem situations by construction appropriate functions.

Use appropriate tools strategically.

- Use technology to enhance understanding of the connections of functions to arithmetic and geometric sequences.
- Use technology to graph equations and to connect to functions.

Look for and make use of structure.

- Use the structure of sequences and series to build related expressions and functions.

Essential questions

- What are the differences and similarities of arithmetic and geometric sequences?
- What is the purpose of function notation?
- Why are functions represented by different models?
- What are the similarities and differences between recursive and explicit functions?

Written Curriculum

Common Core State Standards for Mathematical Content

Seeing Structure in Expressions A-SSE

Write expressions in equivalent forms to solve problems

A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments*.★

Interpreting Functions F-IF

Understand the concept of a function and use function notation

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Building Functions F-BF

Build a function that models a relationship between two quantities [*For F.BF.1, 2, linear, exponential, and quadratic*]

F-BF.1 Write a function that describes a relationship between two quantities.★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Linear, Quadratic, and Exponential Models★ F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students learned understanding of addition and subtraction. During the first grade, students represented and solved problems involving addition and subtraction. They applied the properties of operations and worked with addition and subtraction equations. Second-grade students continued to represent and solve problems involving addition and subtraction and began working with groups of objects to gain foundations for multiplication. In third grade, students represented and solved problems involving multiplication and division. They understood the properties of multiplication and the relationship between multiplication and division. They solved problems involving the four operations and identified and explained patterns in arithmetic. The students developed understanding of fractions as numbers.

Students continued to use the four operations with whole number to solve problems during fourth grade. They gained familiarity with factors and multiples and generated and analyzed patterns. Students extended their understanding of fraction equivalence and ordering. They built fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Students understood decimal notation for fractions and compared decimal fractions. During the fifth grade, students wrote and interpreted numerical expressions and analyzed patterns and relationships. They used equivalent fractions as a strategy to add and subtract fractions. To multiply and divide fractions, students applied and extended previous understandings of multiplication and division. Students applied and extended previous understandings of arithmetic to algebraic expressions during sixth grade. They reasoned about and solved one-variable equations and inequalities. Students represented and analyzed quantitative relationships between dependent and independent variables. Application and extension of previous understanding of multiplication and division was used to divide fractions.

During the seventh grade, students applied and extended previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers. They used properties of operations to generate equivalent expressions. They solved real-life and mathematical problems using numerical and algebraic expressions and equations. In the eighth grade, students knew that there are numbers that are not rational and approximated them by rational numbers. The students worked with radicals and integer exponents. They understood the connections between proportional relationships, lines, and linear equations. They defined, evaluated, and compared functions. Students used functions to model relationships between quantities. In Algebra 1, the students interpreted the structure of expression and wrote expressions in equivalent forms to solve problems. They created equations that derived number or relationships. They solved equations and inequalities. Properties of exponents are extended to include rational exponents.

Current Learning

Students write algebraic expressions and put them in equivalent form to make equations. They write and understand functions written with function notation. Students write the functions from tables, graphs, or written language (linear, quadratic, and exponential functions). Functions are written to demonstrate arithmetic and geometric sequences. Students learn the difference between explicit and recursive functions.

Future Learning

The concepts in this unit will be used in fourth-year as well as college mathematics courses. Students will use the equations here to calculate home mortgages, savings accounts, and credit card payments. Forest, conservation, and logging workers are occupations that use functions as well as arithmetic and geometric sequences. Purchasing managers, buyers, and purchasing agents use the formula to track sales and sales projections. Farmers, ranchers, and agricultural managers use the concepts in the chemical and medications they need for crop growth or animal health. They must also be good business people as discussed previously.

Additional Findings

This unit is of the utmost importance. All the clusters in this unit are on the end-of-course test. Understand the concept of a function and use of function notation (F-IF.3) is a supporting standard in Algebra 2. This standard should support the Major work in F-BF.2 for coherence. Build a function that models a relationship between two quantities (F-BF1,2) is another function. The end of the year course test uses tasks in a real-world context. Tasks may involve linear functions, quadratic functions, and exponential functions..

Algebra 2, Quarter 3, Unit 3.3

Connecting Exponential and Logarithmic Functions Using Multiple Representations

Overview

Number of instructional days: 10 (1 day = 45–60 minutes)

Content to be learned

- Expand exponential properties to rational exponents.
- Interpret functions using graphs and tables.
- Interpret functions graphically and verbally.
- Use different models to write linear and exponential functions
- Determine if a situation is an exponential or a linear function.
- Relate growth and decay function to exponential and logarithmic functions.

Mathematical practices to be integrated

Make sense of problems and persevere in solving them.

- Analyze problem situations involving exponential or logarithmic functions to solve them.
- Explain the correspondences between the graphs of functions, verbal descriptions, and tables.

Reason abstractly and quantitatively.

- Represent a contextual problem situation abstractly.
- Interpret functions expressed graphically or verbally.

Model with mathematics.

- Model linear and exponential functions using multiple representations.
- Model a problem situation with an exponential or a linear function.

Essential questions

- How do you determine if a situation should be written linear or exponentially?
- Why would you use the different representations of a function?
- Why are exponents represented as fractions?
- What purpose does interpretation of a function serve?
- How are growth and decay functions related to exponential and logarithmic functions?

Written Curriculum

Common Core State Standards for Mathematical Content

The Real Number System — N-RN

Extend the properties of exponents to rational exponents.

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want*
$(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Seeing Structure in Expressions — A-SSE

Write expressions in equivalent forms to solve problems

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Interpreting Functions — F-IF

Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Building Functions — F-BF

Build a function that models a relationship between two quantities

F-BF.1 Write a function that describes a relationship between two quantities.☐

c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Linear, Quadratic, and Exponential Models★ F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.★

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★

F-LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

Interpret expressions for functions in terms of the situation they model

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.★

Common Core State Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Clarifying the Standards

Prior Learning

In kindergarten, students learned understanding of addition and subtraction. During the first grade students represented and solved problems involving addition and subtraction. They had to apply the properties of operations and worked with addition and subtraction equations. Second grade students continued to represent and solve problems involving addition and subtraction and worked with groups of objects to gain foundations for multiplication. In third grade, students represented and solved problems involving multiplication and division. They understood the properties of multiplication and the relationship between multiplication and division. They solved problems involving the four operations and identified and explained patterns in arithmetic. The students developed an understanding of fractions as numbers.

Students continued to use the four operations with whole number to solve problems during fourth grade. They gained familiarity with factors and multiples and generated and analyzed patterns. Students extended understanding of fraction equivalence and ordering. They built fractions from unit fraction by applying and extending previous understandings of operations on whole numbers. Students understood decimal notation for fractions, and compared decimal fractions. During the fifth grade, students wrote and interpreted numerical expressions and analyzed patterns and relationships. They used equivalent fractions as a strategy to add and subtract fractions. To multiply and divide fractions, students applied and extended previous understandings of multiplication and division. Students applied and extended previous understandings of arithmetic to algebraic expressions during sixth grade. They reasoned about and solved one-variable equations and inequalities. Students represented and analyzed quantitative relationships between dependent and independent variables. Application and extension of previous understanding of multiplication and division was used to divide fractions.

During the seventh grade, students applied and extended previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers. They used properties of operations to generate equivalent expressions. They solved real-life and mathematical problems using numerical and algebraic expressions and equations. In the eighth grade, students knew that there are numbers that are not rational, and approximated them by rational numbers. The students worked with radicals and integer exponents. They understood the connections between proportional relationships, lines, and linear equations. They defined, evaluated, and compared functions. Students used functions to model relationships between quantities. In Algebra I the students interpreted the structure of expression and wrote expressions in equivalent forms to solve problems. They created equations that derived number or relationships. They solved equations and inequalities. Properties of exponents are extended to include rational exponents.

Current Learning

Students expand the properties of exponents to rational exponents. Functions are interpreted graphically, algebraically, tabularly, and verbally. Students write linear and exponential functions from different models, and they determine if a situation would be best represented by a linear or exponential function. They relate growth and decay equations to exponential and logarithmic functions.

Future Learning

Students will use exponential and logarithmic functions in the fourth-year mathematics course and in college mathematics courses. This content will be useful for engineers, electrical and electronic repairmen and installers, and financial managers.

Additional Findings

None found.

Algebra 2, Quarter 3, Unit 3.4

Using Arithmetic Operations and Graphing with Rational Expressions

Overview

Number of instructional days: 15 (1 day = 45–60 minutes)

Content to be learned

- Write rational expressions in different forms.
- Solve rational expressions following the rules of operations.
- Provide explanations for each step when solving rational equations.
- Identify extraneous solutions.

Mathematical practices to be integrated

Model with mathematics.

- Model problem situations with rational equations.
- Interpret mathematical results of a problem solution in the context of the contextual situation.

Use appropriate tools strategically.

- Use technology to graph rational equations.
- Use a graphing calculator to perform operations on rational expressions.

Look for and make use of structure.

- Explore the structure of rational expressions to interpret the meaning.

Essential questions

- What are the similarities and differences of solving equations and rational equations?
- Why do some equations have extraneous solutions?

Written Curriculum

Common Core State Standards for Mathematical Content

Arithmetic with Polynomials and Rational Expressions A-APR

Rewrite rational expressions [Linear and quadratic denominators]

A-APR.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

*A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning [*Simple ~~radical~~ and rational*]

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.2 Solve simple rational ~~and radical~~ equations in one variable, and give examples showing how extraneous solutions may arise.

**for Algebra II PAP; optional for Algebra II*

Common Core State Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Clarifying the Standards

Prior Learning

In kindergarten, students learned understanding of addition and subtraction. During the first grade students represented and solved problems involving addition and subtraction. They applied the properties of operations and worked with addition and subtraction equations. Second grade students continued to represent and solve problems involving addition and subtraction and worked with groups of objects to gain foundations for multiplication. In third grade, students represented and solved problems involving multiplication and division. They understood the properties of multiplication and the relationship between multiplication and division. They solved problems involving the four operations and identified and explained patterns in arithmetic. The students developed understanding of fractions as numbers.

Students continued to use the four operations with whole number to solve problems during fourth grade. They gained familiarity with factors and multiples and generated and analyzed patterns. Students extended understanding of fraction equivalence and ordering. They built fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Students understood decimal notation for fractions, and compared decimal fractions. During the fifth grade, students wrote and interpreted numerical expressions and analyzed patterns and relationships. They used equivalent fractions as a strategy to add and subtract fractions. To multiply and divided fractions, students applied and extended previous understandings of multiplication and division. Students applied and extended previous understandings of arithmetic to algebraic expressions during sixth grade. They reasoned about and solved one-variable equations and inequalities. Students represented and analyzed quantitative relationships between dependent and independent variables. Students used application and extension of previous understanding of multiplication and division to divide fractions. Students understood ratio concepts and used ratio reasoning to solve problems.

During the seventh grade, students applied and extended previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers. They used properties of operations to generate equivalent expressions. They solved real-life and mathematical problems using numerical and algebraic expressions and equations and analysis of proportional relationships. In the eighth grade, students knew that there are numbers that are not rational, and they approximated them by rational numbers. The students worked with radicals and integer exponents. They understood the connections between proportional relationships, lines, and linear equations. They defined, evaluated, and compare functions. Students used functions to model relationships between quantities. In Algebra I, students interpreted the structure of expression and wrote expressions in equivalent forms to solve problems. They created equations that derived number or relationships. They solved equations and inequalities. Students extended properties of exponents to include rational exponents.

Current Learning

Students write rational expressions in different forms. The various operations are used to solve rational equations. Students understand that rational expressions follow the same rules of operations as whole numbers and fractions. Students provide explanations for each step of solving rational equations. When solving rational equations, students give examples of extraneous solutions.

Future Learning

Students will continue to analyze and graph rational expressions in future mathematical courses. aerospace engineers, chemists, and medical and health services managers are professions that use the skills in this unit.

Additional Findings

None found.